

ALGEBRA II WITH TRIGONOMETRY

Algebra II with Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II with Trigonometry is required for all students pursuing the *Alabama High School Diploma with Advanced Academic Endorsement*. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II with Trigonometry course.

Number and Operations

Students will:

1. Determine relationships among subsets of complex numbers.
Example: using Venn diagrams or tree diagrams to show how subsets of complex numbers are related
2. Use order of operations, conjugates, and absolute value to simplify expressions involving complex numbers.
Examples: Simplify $\sqrt{8}$. Answer: $2i\sqrt{2}$.
Simplify $(4-2i)^2$. Answer: $12-16i$.
Simplify $\frac{3+i}{3-i}$. Answer: $\frac{4+3i}{5}$.

Algebra

3. Determine effects of shifts, reflections, and dilations on families of functions, including $y = \frac{k}{x}$ (inverse variation), $y = kx$ (direct variation/linear), $y = x^2$ (quadratic), $y = a^x$ (exponential), and $y = \log_a x$ (logarithmic).
Example: comparing the graphs of $y = 2^x$, $y = 2^x + 1$, $y = 2^{x+1}$, and $y = -2^x$
 - Identifying the domain and range of a relation given its graph, a table of values, or its equation, including those with restricted domains
Examples: Find the domain of $y = \frac{1}{x-3}$. Answer: $(-\infty, 3) \cup (3, \infty)$.
Find the domain of $y = \sqrt{x-2}$. Answer: $x \geq 2$.
 - Identifying application-based situations corresponding to families of functions

4. Determine the nature of solutions of a quadratic equation.
 Examples: *Use the discriminant to determine the nature of the roots of $x^2 - 3x + 5 = 0$.*
 Answer: no real roots.
 Use the discriminant to determine the nature of the roots of $x^2 + 4x + 3 = 0$.
 Answer: 2 real roots.
 Use the discriminant to determine the nature of the roots of $x^2 + 2x + 1 = 0$.
 Answer: 1 real root.

5. Determine approximate real zeros of functions graphically and numerically and exact real zeros of polynomial functions by completing the square and applying the zero product property and the quadratic formula.
 - Deriving the quadratic formula
 Example: *Approximate real zeros of the polynomial function $f(x) = x^2 + 6x - 10$ on a graphing calculator.*
Answer: $x = -3 \pm \sqrt{19}$.

6. Identify characteristics, including maximum and minimum values, of quadratic functions from their roots, graphs, or equations.
 - Determining a quadratic equation when given its graph or roots
 - Constructing the graph of a function when given its equation
 Examples: graphing equations of the form $y = a(x - h)^2 + k$, graphing equations of the form $y = ax^2 + bx + c$
 - Using the maximum or minimum value of a quadratic function to solve application-based problems

7. Perform operations, including addition, subtraction, multiplication, division, and composition of functions, with polynomial and rational expressions containing variables.
 Example: *Simplify $\frac{3}{x+5} + \frac{5}{x^2 + 6x + 5}$. Answer: $\frac{3x+8}{x^2 + 6x + 5}$.*
 - Determining the inverse of a function or a relation
 - Evaluating rational functions

8. Apply laws of exponents to simplify expressions, including those containing zero and negative integral exponents.
 - Applying laws of logarithms to simplify expressions and solve equations

9. Solve equations, inequalities, and applied problems involving rational and irrational exponents, absolute values, radicals, and quadratics over complex numbers, as well as simple trigonometric, exponential, and logarithmic functions with the solution represented as a graph on a number line, set notation, and interval notation.
 Examples: *Solve $x^2 - 4 > 0$. Answer: $(-\infty, -2) \cup (2, \infty)$.*
 Solve $3^x = 81$. Answer: $x = 4$.

10. Solve systems of linear equations or inequalities in two and three variables using algebraic techniques, including those involving matrices.
 - Calculating the determinant of a 2×2 and a 3×3 matrix
 - Solving two- and three-variable word problems involving application-based situations
11. Determine specific unit circle coordinates associated with special angles.
 - Converting angle measures from degrees to radians and from radians to degrees
 - Graphing angles in standard position
 - Determining the value of the six trigonometric functions for special angles
12. Graph trigonometric functions of the form $y = a \sin(bx)$, $y = a \cos(bx)$, $y = a \tan(bx)$, $y = a \sec(bx)$, $y = a \csc(bx)$, and $y = a \cot(bx)$.
 - Determining period and amplitude of sine, cosine, and tangent functions from graphs or basic equations
Example: solving problems involving harmonic motion
 - Graphing angles in standard position

Geometry

13. Solve coordinate geometry problems using algebraic techniques.
Examples: finding missing coordinates of vertices of polygons, determining properties of polygons given the coordinates of their vertices, determining perpendicularity of the sides of a polygon on a coordinate plane
14. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.
 - Applying the law of sines and the law of cosines to determine missing measures of triangles
15. Verify simple trigonometric identities using Pythagorean and reciprocal identities.
Example: verifying $\cos^2 \alpha + \tan^2 \alpha \cos^2 \alpha = 1$

Data Analysis and Probability

16. Use multiple representations, including graphical, numerical, analytical, and verbal, to compare characteristics of data gathered from two populations.
 - Identifying characteristics of the design of an experimental study
 - Describing effects of an experimental study design on its outcome
 - Predicting population characteristics using sample statistics
 - Identifying characteristics, including the mean and standard deviation, of a normal distribution
17. Analyze data to determine if a linear, quadratic, or exponential relationship exists.
 - Determining an equation of linear regression from a set of data to predict outcomes
18. Calculate probabilities of events using permutations, combinations, the laws of probability, and the binomial theorem.
 - Calculating conditional probability